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HYDRODYNAMICS OF EXPLOSIONS

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The hydrodynamics of explosions, as a significant scientific specialization of the physics and mechanics of explosion processes, encompasses many problems, ranging from the generation and propagation of shock waves to the behavior of media under explosive loads. Their solution also involves the development of new experimental methods, and the creation of mathematical models of the observed effects. The last problem, however, is in many ways simplified, since the wide spectrum of flows arising in this case is described by a quite limited number of models. One of the most widely employed and simplest models is the model of an ideal incompressible liquid. It is successfully employed for the theoretical analysis of many phenomena of a typically explosive character and is based on the real possibilities of neglecting the strength and plastic properties of the media, friction forces, and compressibility under the extremely high pressures generated by the explosive loads. The use of such extremely simplified models often makes it possible to understand the essence of the process, though in making comparisons with experimental data they must also be modified.

This review is concerned with the analysis of the basic results of experimental and theoretical research on the mechanics of explosives, carried out in the Siberian Branch of the USSR Academy of Sciences over a period of 30 years from 1957 to 1986, in three important areas: shock waves in underwater explosions and cavitation, the problems of cumulation and jet flows, and explosive processes in soils.

Many of the studies enumerated below appeared owing to the attention and often the ideas of M. A. Lavrent'ev, which ultimately turned out to be the foundation for the understanding of the phenomena as a whole.

Shock Waves in Underwater Explosions and Cavitation. Cavity Dynamics. One of the most important problems in the study of underwater explosions is the analysis of the dynamics of a cavity with detonation products as a source responsible for the formation and the parameters of explosion-generated shock waves (SW). This problem is also of interest for a wide range of problems of interaction of SW with isolated cavities and an ensemble of cavities, development of bubble cavitation, formation of SW in underwater explosions of charges with a complex shape, etc. These questions were studied in detail at the Institute of Hydrodynamics of the Siberian Branch of the USSR Academy of Sciences from 1960 to 1980 and were associated with the clarification of the fundamental aspects of the effect of the compressibility of a liquid, the symmetry of flow, and the state of the gas in a pulsating cavity.

V. K. Kedrinskii [1-3] was the first to derive in the acoustic approximation a general equation describing the dynamics of a cavity in two-dimensional, cylindrical, and spherical geometries ($\nu = 0, 1, 2$). The result is based on the analysis of a one-dimensional, potential, isentropic flow of liquid, described by the system of equations (acoustic approximation)

$$c_0^{-2}\Phi_{tt} - \Phi_{rr} - \nu\Phi(1-\nu/2)/2r^2 = 0,\quad \Phi_t = -r^{\nu/2}\Omega, \tag{1}$$

where $\Phi = r^{\nu/2}\varphi$; $\Omega = \omega + u^2/2$; $\omega = \int dp/\rho$. From here it follows that in the two-dimensional and spherical cases the system makes it possible to derive exactly and in the case of cylindrical

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symmetry approximately ($\nu = 1$; asymptotic approximation up to the term $\Phi/4r^2$) with the substitution $r^{\nu/2}\,\Omega = G$ the equation

$$G_t + c_0 G_r = 0, \tag{2}$$

indicating that the function $G = r^{\nu/2}\varphi_t$, which is conserved on characteristics diverging with a velocity c_0, is invariant.

This result, according to the Kirkwood-Bethe model [4], can be extended to the case when the disturbance propagates with a velocity $c + v$:

$$G_t + (c + v)G_r = 0. \tag{3}$$

Substitution of the expressions for G in terms of v and ω into (3), taking into account

$$\partial v/\partial r = -(c^{-2}d\omega/dt + \nu v/r),\ \partial\omega/\partial r = -dv/dt$$

under the conditions that at the cavity wall $\omega = \omega_0$, $r = R$, $v = \dot{R}$ makes it possible to construct the generalized equation for the pulsation

$$R(1 - \dot{R}/c)\ddot{R} + \frac{3}{4}\nu\dot{R}^2(1 - \dot{R}/3c) = \frac{\nu}{2}(1 + \dot{R}/c)\omega_0 + R\dot{\omega}_0(1 - \dot{R}/c)/c, \tag{4}$$

where $\omega_0 = \dfrac{nA}{(n-1)\rho_0}\left[\left(1 + \dfrac{p(R) - p_\infty}{A}\right)^{1-1/n} - 1\right]$; $c = c_0\left[1 + \dfrac{p(R) - p_\infty}{A}\right]^{(n-1)/2n}$; $n = 7.15$, $A = 3.05\cdot 10^2$ MPa.

Cylindrical symmetry, in particular, because of the above-noted possibility for constructing only an approximate model, is a special case. Nevertheless, in spite of the complexity of the experiments, where the length of the charge and the velocity of the detonation are finite, it can be used to interpret real axisymmetric flows, for example, within the framework of the methods of independent sections or a point source moving with the velocity of the detonation.

It was shown in [1] that in some formulations it is possible to construct an equation for the pulsations of a one-dimensional cylindrical cavity in an incompressible liquid also. For the dimensionless variables

$$y = R/R_0,\quad r_0 = r/R_0,\quad H_0 = H/R_0,\quad p_0 = p(0)/p_\infty,\quad \tau = t(p_\infty/\rho_0)^{1/2}/R_0$$

with a corresponding transformation of ω_0 they have the form

a) the limit $c \to \infty$ in Eq. (4)

$$(y\ddot{y} + \dot{y}^2)2 - \dot{y}^2/2 = p_0 y^{-2\gamma} - 1,$$

b) explosion at a depth $H \gg R$ in a liquid with a free surface

$$(y\ddot{y} + \dot{y}^2)\ln(2H_0/y) - \dot{y}^2/2 = p_0 y^{-2\gamma} - 1,$$

c) explosion in a cylindrical layer of liquid with $r^2 \gg R^2$

$$(y\ddot{y} + \dot{y}^2)\ln(r_0/y) - \dot{y}^2/2 = p_0 y^{-2\gamma} - 1.$$

There are two basic parameters according to which the calculations can be compared with experimental data on cavity dynamics: the maximum degree of compression or expansion R_0/R_* and the period of the pulsations T, which is traditionally defined, within the framework of an incompressible liquid, as twice the collapse time of an empty cavity τ. From case a) we have

$$\tau = \sqrt{\frac{2}{3}}\,\Gamma(7/6)\,\Gamma(1/2)/\Gamma(5/3) = 1.485,\quad T = 2\tau.$$

If an approximate estimate is employed for the potential $\varphi = r^{-1/2}\Phi(t)$, where $\Phi(t) = 2R^{3/2}\dot{R}$ follows from the kinematic condition $\dot{R} = -\partial\varphi/\partial r|_{r=R}$, an analytical solution can be constructed for an empty cylindrical cavity [1]:

$$\tau = \sqrt{2(1 - y^2)}.$$

The problem of the dynamics of a cylindrical cavity with detonation products was solved by Kedrinskii and Kuzavov [5, 6].

For many practical problems it is important to know the parameters of the pulsation of the cavities with explosion products and gas-filled bubbles behind the SW front (or with a fixed pressure p_∞ at infinity). We shall present the main parameters [3].

1. Incompressible liquid for p_∞ = const

$$(R_0/R_{\min})^{3\gamma-3} = 1 + (\gamma - 1)A_*, \quad \tau = 0.915 R_0 \sqrt{\rho_0/p_\infty}\ (\mathrm{c}), \quad A_* = p_\infty/p_0;$$

for $p_\infty = p_* \exp(-t/\theta)$

$$(R_0/R_{\min})^{3\gamma-3} \simeq 1 + \mu B^2(\gamma - 1)/(1 + \mu B), \quad \sqrt{\mu} = \theta/R_0 \sqrt{\rho_0/p_0}, \quad B = p_*/p_0.$$

2. Compressible liquid [7] (p_∞ = const):

$$R_{\min}/R_0 \simeq \frac{5}{3}\cdot 10^3\tau + 0.025.$$

3. Parameters of the explosion cavity:

For spherical symmetry

$$R^i_{\max} \simeq \left(\frac{3}{4\pi}\alpha_i E_0 Q\right)^{1/3} p_0^{-1/3}\ (\mathrm{m}), \quad \alpha_i = 0.41;\ 0.14;\ 0{,}076,$$

$$T_1 \simeq 1.14\rho^{1/2}(\alpha_i E_0 Q)^{1/3} p_0^{-5/6}\ (\mathrm{sec}),$$

$$R/R_0 = 665(t/R_0)^{0{,}1} \text{ for } 10^{-4} \leqslant t \leqslant T_1/4$$

(ρ in $\mathrm{kg/m^3}$, t in sec, R_0 in cm);

for cylindrical symmetry

$$R^i_{\max} \simeq \left(\frac{1}{\pi}\beta_i E_0 W\right)^{1/2} p_0^{-1/2}, \quad \beta_i = 0.218;\ 0.14;\ 0{,}11,$$

$$T_i \simeq 1.635\rho^{1/2}(\beta_i E_0 W)^{1/2} p_0^{-1},$$

$$R/R_0 = 320(t/R_0)^{0{,}15} \text{ for } 2\cdot 10^{-4}R_0 \leqslant t \leqslant T_1 R_0/6.$$

Here p_0 is the hydrostatic pressure, Pa; E_0 is the heat of the explosion, J/kg; Q and W are the mass of the charge, kg and kg/m; i is the number of the pulsations; and, α and β are the relative fractions of the energy of the explosive remaining in the detonation products.

One of the main characteristics determining the dynamics of a bubble is the state of the gas filling the bubble. Experimental studies by Kedrinskii and Soloukhin, performed in 1960, and Kedrinskii and Pigolkin [8] on the compression of a cavity filled with explosive gaseous mixtures $2H_2 + O_2$ or $2C_2H_2 + 5O_2$ showed that under the action of blast waves in a liquid the bubbles collapse adiabatically. This fact was established from the agreement between the experimentally computed temperature $T_* = T_0(R_0/R_*)^{3\gamma-3}$ at the moment the mixture is ignited and their known ignition temperatures. In an experimental analysis of the dynamics of the form of pulsating gaseous cavities Kedrinskii and Soloukhin [9] found that under the action of SW a bubble collapses asymmetrically with the formation of a cumulative jet, which forms in the process of collapse and causes fragmentation of the bubble.

There are a number of problems associated with the characteristic features of the formation of the structure of the shock-wave field in underwater explosions of explosive charges with a complex shape or accompanying the transformation of a pulsating, spherical, explosion cavity with detonation products in a gravitational field. Here the flow is determined primarily by the ring or toroidal form of the cavity. The equation describing the one-dimensional pulsations of such a cavity was derived by Kedrinskii for an incompressible liquid [10]

$$\rho \ln(8a/R)(R\ddot{R} + \dot{R}^2) = p_0(R_0/R)^{2\gamma} - p_\infty$$

(a is the radius of the ring), employing a special coordinate system, and for a compressible liquid [2]

$$\ln (8a/R)(R\ddot{R} + \dot{R}^2) - \dot{R}^2/2 = \omega_0 + \pi R \dot{\omega}_0/c_0.$$

Calculations using the last equation and comparison with experimental data showed that as the radius of the ring increases the maximum and minimum radii of the cavity and also the energy balance of the explosion asymptotically approach the data for cylindrical symmetry.

Underwater Explosion of a String Charge: Near Zone and Asymptotics. One of the most successful models which enables a complete analysis of the parameters of the SW from the near zone to the asymptotic zone is the Kirkwood–Bethe model [4], developed for problems with spherical symmetry and extended by Kedrinskii [11, 12] to the case of infinitely long cylindrical explosive charges (of other asymptotic models we call attention to [13-15]). This model is based on the assumption that the function G is invariant on the characteristics. The so-called peak approximation, which takes into account the experimental fact that the pressure behind the SW front decays exponentially, is studied; the parameters are determined only in the region close to the front, which makes it possible to replace in the analysis of the flow the mass velocity behind the front v by the Riemann variable $\sigma = \int_{\rho_0}^{\rho} c\,d\rho/\rho$, and to express the enthalpy on the contact boundary in the form of an exponential function $\omega(t) = \omega(0)\exp(-t/\theta_0)$. The latter makes it possible, if necessary, to avoid the question of the equation of state of the detonation products. The method of solution is as follows.

The condition of invariance of the function G is extended to the case of characteristics converging in the detonation products (denoted by the index *) with a velocity c − v

$$G_t - (c_* - v_*)\, G_r = 0. \tag{5}$$

From (3) and (5) we obtain the initial value of the decay constant for the pressure for any one-dimensional flow within the framework of the peak approximation taking into account the fact that $d\omega = dp/\rho$ and therefore $dp/dt = -\omega(0)\theta_0\rho(0)$:

$$\theta_0 = \frac{2\omega(0) R_0 (\rho c + \rho_* c_*)}{\nu \rho_* c_* c (\alpha_2 - \alpha_1)},$$
$$\alpha_1 = [(c_* - u)(\omega_* + u^2/2) - 2c_* u^2]/(c_* + u),$$
$$\alpha_2 = [(c + u)(\omega + u^2/2) - 2cu^2]/(c - u).$$

For the coordinate of the SW front we obtain the equation

$$t_* + \int_{R(t_*)}^{r_f} dr/(c + v) = \int_{R_0}^{r_f} dr/U_f, \tag{6}$$

in which the integral on the left determines the delay time of the propagation of the disturbance τ. It can be easily found, if the main functions of the problem are expressed in terms of σ:

$$G = r^{\nu/2} c_0 \sigma (1 + \beta\sigma),\ U_f \simeq c_0(1 + \beta\sigma),$$
$$c + v = c_0(1 + 2\beta\sigma),\ \beta = (n + 1)/4c_0.$$

Then for $4\beta\Omega/c_0 \ll 1$ and small R/r we obtain the first nonlinear correction for the acoustics for spherical symmetry

$$\tau_{sp} \simeq (r - R)/c_0 - \frac{2\beta G}{c_0^2} \ln (r/R)$$

and for cylindrical symmetry

$$\tau_c \simeq (r - R)/c_0 - \frac{4\beta G}{c_0^2} (r^{1/2} - R^{1/2}).$$

The expressions for τ indicate the profile of the wave changes as the wave propagates.

From the solution of (6) we obtain the coordinate of the front r_f, which makes it possible to determine the pressure in the front

$$p_f = A\left\{\left[\frac{2}{n+1} + \frac{n-1}{n+1}\sqrt{1 + \frac{4\beta\Omega(t_*)}{c_0}(R/r_f)^{1/2}}\right]^{2n/(n-1)} - 1\right\}$$

and the characteristic time scale ζ for the decay constant $\theta = \zeta\theta_0$, determining the degree of "spreading" of the wave:

$$\zeta = 1 - \frac{\sigma}{c_0(1+2\beta\sigma)} - \frac{2\beta}{c_0^2}\left.\frac{\partial G}{\partial t}\right|_{t=t_*}\int\limits_{R(t_*)}^{r_f}\frac{r^{-1/2}dr}{(1+2\beta\sigma)^3}.$$

The time t_* is "rigidly" tied to the explosion cavity. The results of numerical studies for PETN, trotyl, and hexogen can be generalized by the following dependences for the velocity of the SW front:

$$U_f = 3{,}67 r_f^{-0{,}28}\,\text{km/sec} \quad \text{for} \quad 1 \leqslant r_f \leqslant 10,$$
$$U_f/c_0 = 1 + 0{,}28\left[1 + \left(0{,}16 - 1{,}49\cdot10^{-3}r_f + 6{,}23\cdot10^{-6}r_f^2 + 6{,}54\cdot10^{-8}r_f^3\right)(r_f - 10)\right]^{-1} \quad \text{for } r_f \geqslant 10.$$

Here r_f is scaled to the radius of the charge R_0. The time dependence of the coordinate of the front relative to the dimensionless time $\tau_* = tc_0/R_0$ is given by the relations

$$r_f = (1 + 3{,}135\tau_*)^{0{,}78} \quad \text{for} \quad 0 \leqslant \tau_* \leqslant 5{,}75,$$
$$r_f = 2{,}52\tau_*^{0{,}788} \quad \text{for} \quad 5{,}75 \leqslant \tau_* \leqslant 26{,}3,$$
$$r_f = 1{,}86\tau_*^{0{,}88} \quad \text{for} \quad 26{,}3 \leqslant \tau_* \leqslant 263.$$

For the far field ($r_f > 10^3$), where $G \simeq r^{1/2}p$, $c + v \simeq c_0\left(1 + \frac{n+1}{2nA}p\right)$, $U_f \simeq c_0\left(1 + \frac{n+1}{4nA}p\right)$, the asymptotic solution is obtained as follows. The profile of the SW $p_1 = p_0\exp(-\tau_1/\theta_1)$ is given on some surface r_0. Here τ_1 and θ_1 are taken relative to c_0/R_0. The equation for the time τ at which the quantity $p_1 r_0^{1/2} = \rho_0 G(\tau_1) = pr^{1/2} = \text{const}$ arises at the point r has the form

$$\tau = \tau_1 + \int\limits_{r_0}^{r} dr\Big/\left(1 + \frac{n+1}{2nA}p\right) \quad \text{or}$$

$$\tau = \theta_1\ln\left(p_0 r_0^{1/2}/pr^{1/2}\right) + r - r_0 - \frac{n+1}{nA}pr^{1/2}\left(r^{1/2} - r_0^{1/2}\right) + (n+1)^2p^2r\ln(r/r_0)/4n^2A^2, \tag{7}$$

where τ_1 is defined as $\theta_1\ln(p_0/p_1)$, while p_1 is determined based on the conservation of G. The relation (7) is also valid for $r = r_f$ and corresponds to the time τ_f at which the disturbance $\rho_0 G(\tau_1)$, arising at the time τ_1 on the surface r_0, overtakes the front. If, based on the definition of the velocity of the SW front $dr_f = [1 + (n+1)p/4nA]\,d\tau_f$, τ_f is eliminated from (7), a differential equation relating the amplitude and the coordinate of the front is obtained. It is not difficult to find an analytic expression for $\theta = \tau - \tau_f$. For the initial conditions $p_0 = 4.65$ MPa, $\theta_1 = 59$ and $r_0 = 3200$, the asymptotic behavior in the region $3200 \leq r_f \leq 10^6$ is determined by the relations

$$p = 1370 r_f^{-0{,}705}\ \text{MPa}, \quad \theta = 8{,}4 r_f^{0{,}214}.$$

If $r > 10^6$, then $p \sim r_f^{-0.75}$, while $\theta \sim r_f^{0.25}$, which agrees with the asymptotics determined by L. D. Landau and S. A. Khristianovich [14, 15].

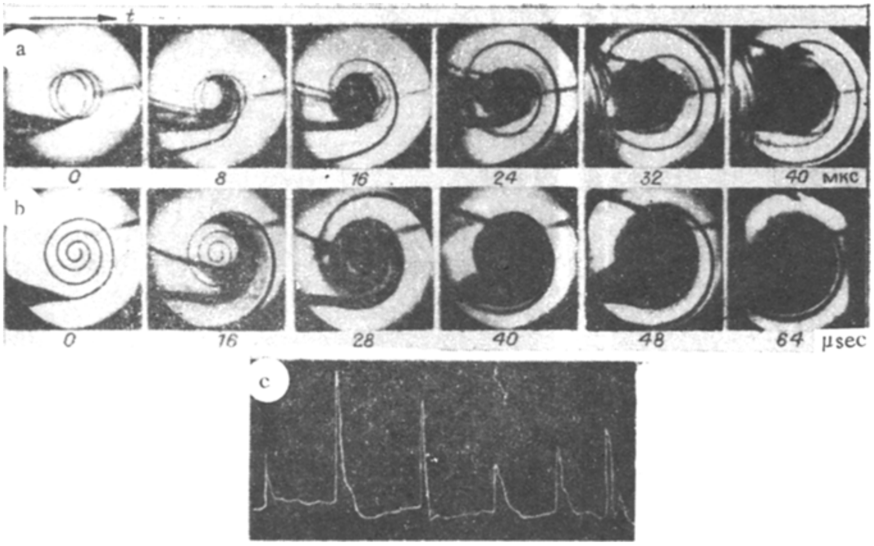

Fig. 1

Explosion Hydroacoustics. The linear string charge and its separate modifications [16], which enable predominant propagation of SW in a plane perpendicular to its axis, are typical examples of so-called explosive sources of sound, widely employed for a wide range of problems in hydroacoustics. In the 1960s and 1970s there was definite interest in sources of the spatial spiral type (for example, [17, 18]) consisting of a high-explosive string charge, radiation from which is intense and directed and has a long duration as well as tonal coloring. The results of investigations of the parameters and structure of the wave field generated by spatial and two-dimensional spiral charges (Figs. 1a and b), carried out in the indicated period independently of Kedrinskii, were generalized in [19]. They showed that the radiation from such sources consists of a packet in the form of a sequence of SW, whose amplitudes are determined from data for concentrated charges with a mass equivalent to the mass of the explosive charge from each loop of the spiral. Figure 1c shows the wave packet recorded at a distance of 20 m on the axis of a two-dimensional spiral; the full sweep equals 5 msec. The repetition frequency of the SW in such systems is determined by the loop length and the detonation velocity, while the duration of the packet is determined by the time required by the detonation front to traverse the entire length of the charge. It is pointed out in [19] that the structure of the wave packet can be substantially changed by changing its duration. Indeed, if the velocities of the SW front in the direction of the axis of the spatial spiral and the axial component of the detonation velocity of the charge are identical, the packet transforms into one long wave, modulated in amplitude with the rotational frequency of the detonation front along the ring elements of the charge.

Laboratory Methods for Generating SW and Their Application. The main source of SW in a liquid is the detonation of an explosive charge. This method is not accessible for laboratory studies [20] and does not permit varying the parameters of the wave within adequate limits. Classical diaphragmed shock tubes [21] and electric discharge methods [22] actually restrict the upper limits of the amplitudes. At the beginning of the 1960s, Vorotnikova, Kedrinskii, and Soloukhin [23] proposed and implemented a new method for generating strong SW, based on the collision of a moving liquid piston with the closed end of a shock tube or with a liquid mass at rest in it. The experiments showed that the method yields SW with a profile of the "step" type and amplitudes in the range 10^2-10^3 MPa and a regulatable duration of the order of 10^{-4}-10^{-3} sec.

If after the moving liquid (state 4) collides with the stationary liquid (state 1) SW propagate on both sides (states 2 and 3), then their amplitude will be determined by the following system of equations

$$p_2 - p_1 = \rho_1 \frac{u^2}{1 - \rho_1/\rho_2}, \quad p_3 - p_1 = \rho_1 \frac{(u_4 - u)^2}{1 - \rho_4/\rho_3}.$$

Because of the low strength of piezoceramic pressure gauges [24] the development of a shock tube of this type required the development of new methods for recording high pressures. Kedrinskii, Soloukhin, and Stebnovskii [23, 25] proposed for this purpose and studied the first semiconductor pressure gages based on germanium, whose volume conductivity (or the conductivity of the contact with the p–n junction) depends strongly on the pressure. A change in the gap width E, equal to about 0.1 eV at 10^3 MPa and leading to an approximately order of magnitude change in the carrier density, which is an exponential function of E, makes the main contribution to this effect.

The method of SW, generated as a result of liquid–liquid collisions, was employed by Kedrinskii, Serdyuk, Soloukhin, and Stebnovskii [26] to determine the velocity and magnitude of the shift in the thermodynamic equilibrium for reversible chemical transformations in solutions under the action of a temperature jump $T - T_0 = 2.6\rho p\cdot 10^{-2}$ (about 3°C at 10^2 MPa). The shift in the equilibrium was recorded based on the change in the concentration of the component of the reaction with the highest absorption coefficient. The parameters of the system made it possible to study the kinetics of fast reactions with a half-transformation time of 2-600 μsec.

Short Waves and the Effect of Bounding Surfaces. In real situations one-dimensional flows are realized, as a rule, only in a zone relatively close to the charge, away from which the effect of the free surface and the bottom becomes determining, and the distortion of the wave profile is significant. The latter is a result of nonlinear effects, consisting of the fact that the unloading wave from the free surface, propagating behind the SW front with a velocity c + v, overtake and attenuate it. This happens when the angle of incidence of the wave on the free surface, measured from the vertical to the surface, is less than some critical value $\theta_* = \sqrt{(n + 1)p_f/2nA}$, which determines the moment at which the irregular reflection occurs [27].

It was possible to describe the characteristic features of the structure and parameters of the SW interacting with interfaces within the framework of the theory of short waves, constructed by Khristianovich, Grib, and Ryzhov [28]. This model is based on the assumption that the quantities characterizing the disturbed motion vary significantly only in a small region adjacent to the SW front. In practice the asymptotic behavior for spherical SW was determined based on this approach in [15]:

$$r_f = \frac{A_0}{p_f}\left[1 + \frac{n+1}{2}\frac{p_f}{nA}\ln(p/p_f)\right] \Big/ (\ln(p/p_f) - 0{,}5)^{1/2},$$

$$0 = \frac{n+1}{2nA}A_0(\ln(p/p_f) - 1)/(\ln(p/p_f) - 0{,}5)^{1/2},$$

where the constants A_0 and p are determined, based on existing experimental data, from curves of the pressure at some distance from the charge.

Lugovtsov [29] showed that the application of this model to the description of wave propagation in a shallow reservoir with depth h_0 leads to practically identical asymptotic behavior for linear and spherical explosive charges:

$$p_f \sim (h_0/r_f)^2.$$

The class of exact particular solutions of the equations for "short waves" was constructed by Zaslavskii [30], who also studied, based on them, the problem of irregular reflection of SW from a free surface (cylindrical symmetry was studied in [31]). Here, in particular, equations for determining the lower boundary of the zone of disturbed flow were derived.

The interaction of SW with the bottom leads, depending on its mechanical properties, to the formation of an entire range of disturbances, preceding the direct wave and the wave reflected from the soil. Here there is also the case of irregular reflection, leading to the formation of a so-called precursor in front of the wave front. The problem of the reflected wave for the case when the soil is represented as an elastic half-space was studied by Shemyakin and Markina [32]. Their solution contains the reflection coefficient $k(\alpha)$, which depends only on the acoustic properties of the soil and the angle of incidence α. The construction of the diagram $k(\alpha)$ makes it possible to determine the zone of irregular reflection and the critical values of α_*, for which the soil behaves analogously to a free surface ($k < 0$).

Cavitation. It has been established experimentally that cavitation, a zone of vapor-gas bubbles, develops near a free surface in the region of regular reflection of SW generated by an underwater explosion. It arises under the action of intense tensile stresses in the region of the training front of the SW, formed as a result of reflection. The parameters of the rarefaction wave were usually determined based on the principle of superposition of waves from real and imaginary charges taking into account the corresponding delay times. The concept of critical values of the tensile stresses, determining the dynamic rupture strength of the liquid, based on which the zones of cleavage as a result of the rupture of the liquid near the free surface were determined, was introduced. This static approach, however, could not explain the profound inconsistencies in the experimental data on the dynamic strength, which was recorded only at the moment at which the visible cavitation bubbles appeared, and led to theoretical estimates which differed substantially from the recorded experimental values. The fact that an insignificant amplitude of the tensile stresses was recorded in a region where there is no visible cavitation could not be explained at all.

Kedrinskii [33, 34] called attention to the fact that in a real liquid there always exists free gas in the form of cavitation nuclei (with radii of the order of $R_0 \cong 10^{-2}$-10^{-5} cm and a volume concentration of $k_0 \cong 10^{-8}$-10^{-12}), and he proposed that in problems of propagation of rarefaction waves the liquid be regarded as a two-phase medium. This enabled him to describe first the dynamics of the development of the cavitation zone from nuclei [33] and to calculate the profile and parameters of rarefaction waves [33, 34] on the basis of the quite simple approximate mathematical model which he proposed:

$$\Delta\zeta = \zeta, \tag{8}$$

$$\frac{d^2k}{dt^2} = -(\alpha k^{1/6})^2(\rho_0 k_0)^{-1}\zeta + \left(\frac{dk}{dt}\right)^2\Big/6k,$$

where $\alpha = (3k_0/R_0^2)^{1/2}$; $\zeta = p - p_0 k^{-\gamma}$, and the spatial coordinate has a scale factor of $\alpha k^{1/6}$.

Numerical investigations of the system (8), carried out for an explosive charge with a mass of 1 g at a depth of 5.3 cm (ten radii of the charge) from the free surface, showed that in the developing cavitation zone the volume concentration of bubbles k grows rapidly with time, reaching values of 10^7-10^9 relative to the initial value k_0 (it was assumed that $R_0 = 5 \times 10^{-5}$ cm, $k_0 = 10^{-11}$). In constructing the computed cavitation zone at different times the fact that the apparatus and methods employed in the experiment have a finite resolution was taken into account. This means that cavitation bubbles which do not reach detectable (smallest visible) sizes or which in the dynamic process are transferred into the lower (relative to this threshold) part of the spectrum, cannot be recorded. From here there arises the concept of the lifetime of a visible bubble and correspondingly the dynamics of the visible zone of cavitation. Comparison with experiment shows that the model (8) describes this effect quite well [33]. Figure 2 shows for one point on the axis of symmetry the dependences k(t) and p(t) for the case indicated above. The broken line in the graph of k(t) separates the cavitation zone into visible and invisible zones, while the points at which the broken line intersects the curves k(t) determine the time interval in which the visible size of a bubble exists at the given point of the computed region. Profiles of rarefaction waves are shown at the bottom. One can see from a comparison of the graphs that it is meaningless to record the dynamic load at the moment at which the bubbles reach visible size, since by this moment the tensile stresses in the medium have practically vanished and to record their maximum (curves 1) a time resolution on the graph of the order of 10^{-9}-10^{-8} sec is required.

Calculation of rarefaction waves made it possible to observe one other important fact. It turned out that the maximum negative pressures recorded in a liquid depend not so much on the maximum amplitudes of the tensile stresses as on the time at which they are applied. If the maximum amplitude appears instantaneously, it can be recorded. However the time during which the medium preserves it is short: by 10^{-7} sec the unloading virtually vanishes (for a single-phase liquid this time is two orders of magnitude longer, 10^{-5} sec). If the slope of the rarefaction-wave front reaches 10^{-6} sec, the maximum tensile stresses admitted by the cavitating liquid decrease by two orders of magnitude (curve 3). It is also interesting to note that the unloading vanishes even before the bubbles reach visible size — approximately by 7 μsec. The curves 2 were obtained with a front slope to 0.1 μsec.

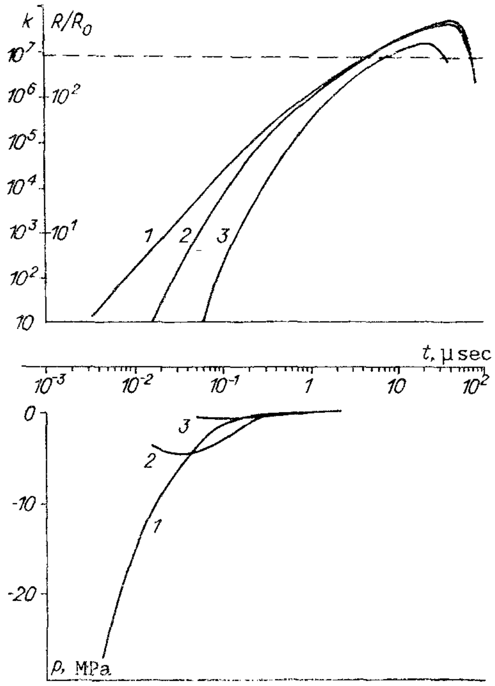

Fig. 2

Fig. 3

Experiments in two-dimensions, carried out by the author, showed that when a strong SW is reflected from a free surface the liquid near it breaks down, forming some sequence of flat layers (Fig. 3), cleavages, each of which in its turn consists not of a continuous, but rather cavitating liquid.

The process of breakdown of the liquid in unloading waves, as one can see, is complicated and develops in stages: growth of cavitation nuclei and transfer of the liquid into a two-phase state, unbounded growth of cavitation bubbles in the zone and formation of a foam structure, breakdown of the foam and formation of "boiling" cleavages. This type of development of cavitation, whose characteristic times are much longer than the duration of the applied tensile stresses, is called irreversible breakdown of the liquid, in contrast, for example, to the phenomenon of ultrasonic cavitation, when the zone vanishes when the source generating it is switched off.

It is logical to assume that there exists some energy threshold, characterizing the possibility of irreversible breakdown of a unit mass of liquid in the rarefaction zone. Stebnovskii and Chernobaev [35] established experimentally that when a liquid volume is broken down by an exploding wire, the threshold at its center is of the order of 1 J/g. Naturally, this integral characteristic is not absolute: it can depend on the pressure gradient behind the front of the SW incident on the free surface, and irreversible breakdown can effect only part of the volume loaded by the explosion. Stebnovskii [36] noted that the process of breakdown is accompanied by a complicated pattern of development of disturbances on the outer boundary of the liquid volume, which is determined by some dimensionless parameter, characterizing the ratio of the inertial and capillary forces.

The two-phase model of the development of bubble cavitation for determining the structural dynamics of the wave field in a liquid was successfully applied, in an exact formulation, by Kedrinskii and Plaksin to solve a number of classical problems. In the problem of the generation of ultrasonic waves [37, 38] it was shown that as the frequency of the oscillations of the piston decreases the maximum negative pressures supported by a real cavitating liquid are much lower, while the profile is distorted to such an extent that one cannot talk about regularity of pressure oscillations. It was found that the unloading wave, calculated in [39] within the framework of the classical shock tube, has a fine structure. It turned out that under conditions of outflow of a preloaded liquid volume, there forms in it a perturbed flow, which separates into a precursor 1, formed by a centered rarefaction wave and propagating with the frozen velocity of sound, and the main disturbance — a rarefaction wave 2 (Fig. 4a), whose oscillating profile flattens out with time and approaches a monotonic profile. The structure of the SW reflected from the free surface turned out to be nontrivial

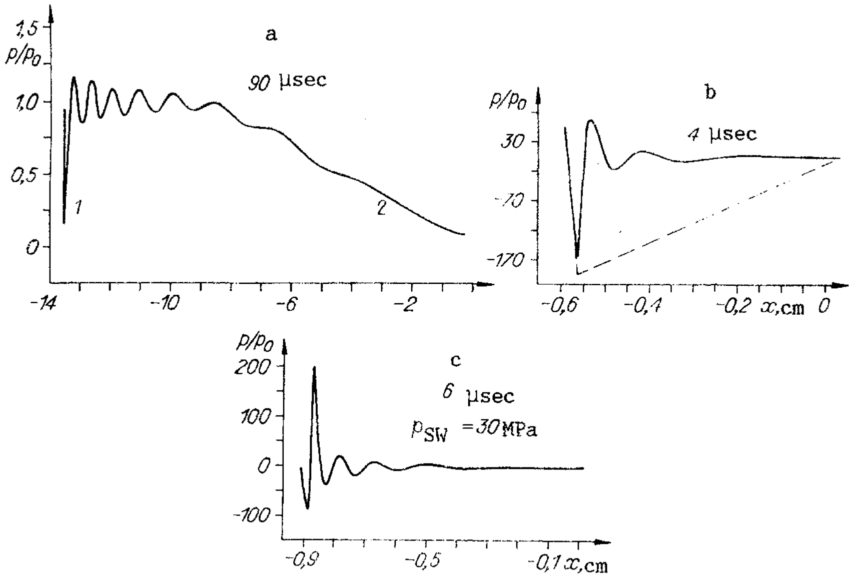

Fig. 4

[40]; it transforms into a wave packet (flat one-dimensional formulation) with a high amplitude in the positive phase (Figs. 4b and c); the broken line shows the profile of the rarefaction wave for the single-phase model.

Kedrinskii, Merkh, and Khansson were the first to apply the two-phase model to the analysis of the pressure field in problems of erosion testing [41], when the zone of intense cavitation develops in a narrow gap between the ultrasonic radiator and a stationary sample or near the bottom of a tube with liquid, to which a downwards acceleration of the order of 10^3 g is imparted within tens of microseconds. This problem was studied numerically for a polydispersed composition of cavitation nuclei by Kedrinskii, Pederson and Khansson [42], who showed that the expansion of the size spectrum of the bubbles removes the singularity associated with the appearance of unrealistically narrow pressure peaks with high amplitude, correlated with the moment at which the cavitation cluster collapses.

Uncertainty of the initial state of the cavitation nuclei is characteristic for all cavitation problems mentioned above: their size spectrum and the "partial" density. This is also linked with the problems of resolving the physical state of the liquid, loaded by the rarefaction wave, and the understanding of the mechanism of development of bubble cavitation. In recent years fundamental results have been obtained in this area at the Institute of Hydrodynamics of the Siberian Branch of the USSR Academy of Sciences. Besov, Kedrinskii, and Pal'chikov, employing a method based on the diffraction of a laser beam by micrononuniformities and the dynamics of the scattering function, were able to observe and prove experimentally that in distilled water the monodispersed composition of gas nuclei with radii of about 1.5 μm predominates [43]. The density of micrononuniformities, evaluated from photographs of tracks of their diffraction spots in the region of liquid illuminated with a laser beam, turned out to be of the order of 10^5 cm^{-3}.

Based on the results of the analysis of experimental data and theoretical investigations, Kedrinskii proposed a new model for the formation of dense bubble clusters [42], based on the dependence of the time at which the microbubbles reach a detectable size on their starting spectrum and the parameters of the wave: 1) the starting density of micrononuniformities is already of the order of 10^5-10^6 cm^{-3}; 2) in the region of threshold values of the applied stresses the cluster is saturated with bubbles gradually owing to systematic growth to a visible size of nuclei from the bottom part of the spectrum; and 3) for much larger stresses "instantaneous" saturation of the zone with bubbles, whose entire starting spectrum reaches visible sizes simultaneously, occurs. Analysis carried out by the author together with Plaksin and Kovalev for the example of the dynamics of a single bubble [44] and together with Pederson for the cavitation zone [42] confirmed the reality of the proposed mechanism for multiplication of detectable bubbles in a cluster.

Shock and Sound Waves in Bubble Media. A series of experimental and theoretical investigations of the propagation of strong SW in a liquid with gas bubbles, which made it possible to establish the basic laws governing the process, the mechanism for the transformation of

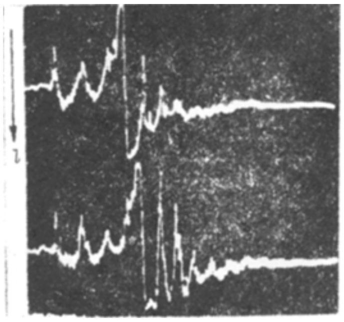

Fig. 5

the energy of the SW, attenuation of the SW, and the formation of the structure, was performed at the Institute of Hydrodynamics in the 1960s. The characteristic features of this process are determined by the nonequilibrium nature of the liquid and gas phases with respect to the pressure and the complicated character of the absorption and reradiation of the energy of the wave by the two-phase medium. Experiments on the analysis of the attenuation of waves in bubble screens with different acoustic properties were begun by Minin, who examined also the first convenient model of a screen — a sequence of alternating flat one-dimensional liquid and gas layers. Some characteristic features of this model were studied in [45] from a different viewpoint.

The first complete experimental analysis of the transformation of SW in bubble media was performed by Kedrinskii [46], who discovered that as a short wave penetrates into a bubble layer it separates into a precursor, propagating with the velocity of sound in the liquid, and the main disturbance, "traveling" in the form of a wave packet with a much lower equilibrium velocity. It is shown in [45] that as the thickness of the bubble layer ℓ increases successive formation of several precursors can be observed (Fig. 5). In [46] a similarity criterion was found for the attenuation of the amplitude of a shock wave in the layer

$$\eta = (3k_0)^{1/2}\, l/R_0 = \Omega l/c_*.$$

Here Ω is the characteristic frequency, and c_* is the equilibrium velocity of sound. Numerical studies of the transformation of a short SW by a bubble layer and propagation of a wave in the half-space occupied by gas bubbles were also performed there within the framework of an approximate model [the system of equations (8)]. It was found that strong dynamic loads (amplification of the wave) appear under conditions of pulsation on the solid wall of the bubble layer, initially located in a region of depressed pressure. It turned out that when the hydrostatic pressure p_0 is abruptly restored the pulsations of the bubbles and strong inertial effects lead to the generation of a series of strong pulses with amplitudes of $(10\text{-}80)p_0$, depending on the degree of the initial rarefaction, over the entire surface of the wall.

It is well known that bubble media are characterized by dispersion. Fox, Carley, and Larson [47] observed in their experiments on the attenuation of sound in a medium with a low volume content of gas bubbles that there is no "window of nontransparency" in the dispersion dependence. Kedrinskii showed for the first time in [46], on the basis of a two-phase model, that the "bleaching" effect is linked not with dissipative losses, but rather with the polydispersed size spectrum of the bubbles, whose partial concentrations varied in the range 0.00015-0.00025.

Problems of Cumulation and Jet Flows. A number of fundamental results in the area of fundamental research on the mechanics of explosions were obtained at the Institute of Hydrodynamics of the Siberian Branch of the USSR Academy of Sciences. This is not surprising, since the school created by Lavrent'ev for the physics and mechanics of explosion processes, based on the models which he formulated, developed under his leadership and direct participation new directions in this area. In what follows we shall analyze the results concerning only cumulative jet flows and effects associated with them.

Classical Cumulation. The understanding of the mechanism of jet formation accompanying compression of the products of an explosion of metallic cones (linings) and the piercing of armor by them was developed by Lavrent'ev [48], who proposed that under explosive loads the

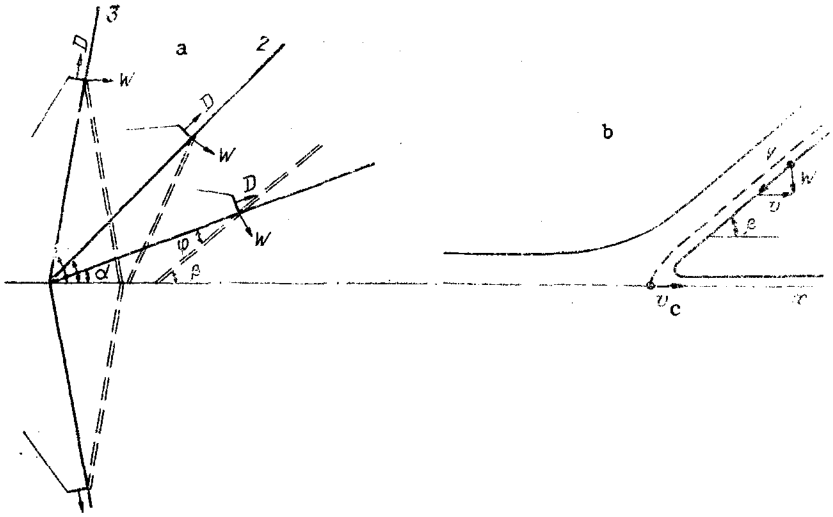

Fig. 6

metal be regarded as an ideal incompressible liquid and reduced the problem to the classical theory of stationary jet flows.

A diagram of the structure of a cumulative jet flow is shown in Fig. 6. In the system of the contact point (a), moving from left to right with a velocity v_c, it is viewed as a jet, flowing from infinity with a velocity V against a wall. In the laboratory system the velocity of the elements of the lining is determined by the vector W, making angle $(\pi - \varphi)/2$ with the surface of the lining (in Fig. 6b it is indicated by the double broken line). Here φ is the angle of instantaneous rotation of the lining by the sliding detonation wave. From the figure one easily find that

$$v_c = W\cos(\varphi/2)/\sin\beta = D\sin\varphi/\sin\beta,$$
$$V = W\sin(\varphi/2 + \beta)/\sin\beta = D(1 - \sin\alpha/\sin\beta).$$

From here the velocity of the jet and rammer have the form

$$v_{jet} = v_c + V = W\frac{\cos(\alpha/2)}{\sin(\beta/2)} = D\left(1 + \frac{\sin[(\varphi-\alpha)/2]}{\sin(\beta/2)}\right),$$
$$v_{ram} = v_c - V = W\frac{\sin(\alpha/2)}{\cos(\beta/2)} = -D\left(1 - \frac{\cos[(\varphi-\alpha)/2]}{\cos(\beta/2)}\right).$$

The structure of the flow for $(\alpha + \varphi) < \pi/2$ has been well studied; this is a system consisting of a high-speed cumulative jet with a relatively high specific kinetic energy and a so-called low-velocity rammer, containing the main part of the mass of the starting lining. Their masses are related by the relation $m_{jet}/m_{ram} = \tan^2(\beta/2)$.

Titov [49] called attention to the possibility of the existence of a regime of so-called "reverse" cumulation, characteristic for shallow linings (the angle $\alpha + \varphi \gtrsim \pi/2$).

The turning angle φ obviously depends primarily on the physical parameters of the problem: the lining material and the type of explosive, the ratio of their masses, etc. If it is fixed and the angle α is increased (Fig. 6b, curves 1-3), then it is easy to see that the velocity vector W is oriented away from the axis of symmetry — the cumulation effect decreases, and then there arises a reversal of the flow relative to the standard flow, associated with the unique "slipping through" of the lining. The mass of the jet along the x axis increases substantially, and although in the optimal regime its velocity drops by a factor of 1.5-2, the specific energy of the matter, as before, is several times higher than the specific energy of the explosion.

Gorshkov [50] measured experimentally the velocity distribution along the jet for the reverse regime, and employing a well-known method he reproduced the dynamics of the angle between the converging lining and the core, and showed that the jet forms partly by the classical scheme and partly by the reverse scheme.

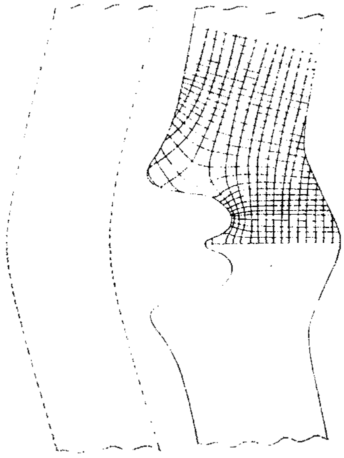

Fig. 7

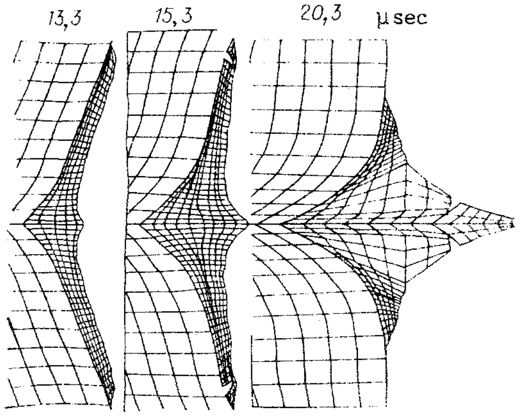

Fig. 8

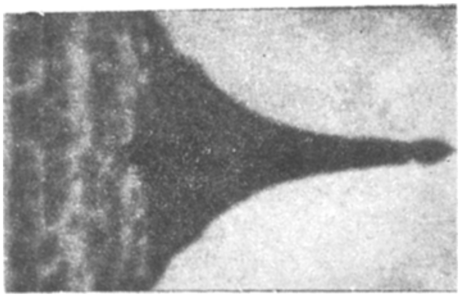

Fig. 9

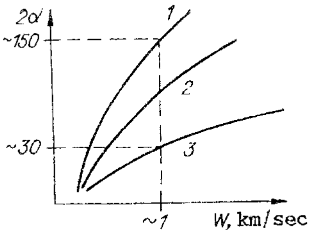

Fig. 10

Kedrinskii and Stebnovskii calculated in a two-dimensional impulsive formation the nonstationary starting stage of the formation of the structure of a cumulative flow for shallow linings. The lining was regarded as a strip of ideal incompressible liquid, bent at an angle 2α, and the effect of the detonation products was modeled by a special starting distribution of the pressure pulse along the outer boundary of the strip. Figure 7 shows the computed form of the lining at the time $t = 25.8$ μsec (solid line). One can see that at the center of the strip the flow is reversed, an inverse cone is formed and from the cone, as a result of compression, there forms a reverse cumulative jet, directed toward the starting position of the lining (broken line), and a rammer, as the main element of this regime of the flow. Its velocity equals 1.32 km/sec (a lead cone with an angle of $2\alpha = 150°$, a thickness $\delta = 2$ cm, and a base radius of ∿17 cm was studied). The starting stage of reversal is already recorded at $t = 3.7$ μsec.

Merzhievskii and Resnyanskii calculated, within the framework of a model of a visco-elastic body of the Maxwellian type [51], the collapse of a shallow (120°) copper lining, taking into account its strength properties. The dynamics of the process of formation of the shell is shown in Fig. 8. Based on an analysis of the dynamics they assert that at first the process develops according to the standard scheme and emerges into the new regime only after some time. The figures presented give a quite ideal representation of the angle between the converging elements and the axis, based on which it is difficult to draw an unequivocal conclusion about the characteristic features of the structure of the flow. Judging from the dynamics of the outer form, one gets the impression that the flow develops primarily according to the "classical scenario" while the reverse cumulation effect is a consequence of complex deformation with breakdown and "inversion" of the periphery of the lining and not its center. These calculations can be compared with the experimental x-ray diffraction picture, obtained by Titov, of one of the intermediate stages of the deformation of the lining (Fig. 9).

The hydrodynamic theory, as pointed out in [48], has substantial limitations. They are manifested primarily at small collision angles. According to [52], jets arise only if the collapse of the lining (and the motion of the point of contact) occurs with subsonic velocity (in Fig. 10, below curve 3, the flow is not a jet flow). Trishin and Kinelovskii [53]

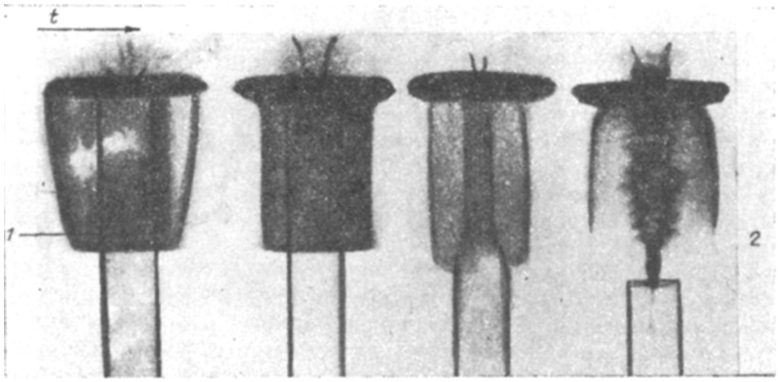

Fig. 11

showed that in the region of the curves 3 and 2 (Fig. 10) the jet is unstable and is fragmented. The maximum possible velocity of monolithic jets is determined by the expression $V_{c,max} = c_0 + \sqrt{c_0^2 + W^2}$.

The discrepancy between the experimental and computed data for a number of basic characteristics of the process, such as the velocity of the cumulative jets, the deformation of the indicator line, playing an important role in the estimates of the structure of the flow under conditions of oblique collisions, and other characteristics, have made it necessary to study viscous models of the lining [54–57], which, however, do not eliminate the existing discrepancies. Rubtsov, who proposed a model of a "pseudoplastic liquid," was able to achieve good agreement with experiment [58]. He introduced the similarity parameter $B = \sqrt{3}\rho W^2/\sigma_T(T)$, which determines the ratio of the pressure at the critical point to the yield stress of the metal. In the limit $B \to \infty$ one obtains the flow of ideal liquid, and for $B \gg 1$ the flow is qualitatively similar to the case of a viscous liquid because of the existence of vorticity boundary layers near free boundaries. Obviously the dynamic yield stress determines the relative fraction of the energy of the system expended on overcoming the strength forces and, therefore, the possibility of jet formation for real metals. The latter is evaluated from the critical collision velocity W_*, for which this process terminates: $W_* = \sqrt{2\sigma/\rho}/\cos\alpha$. This relation determines the curve 1 in Fig. 10, which bounds the region of jet formation on the left [53].

Dynamic problems of the behavior of axisymmetric linings under the action of explosive loads play an important role in understanding the formation of cumulative flows. As a rule, they are studied approximately within the framework of the method of independent sections, when the problem reduces to one-dimensional collapse of a cylindrical ring, and then by taking into account the time delays the picture of the axisymmetric flow is reconstructed. This approach was employed by Kinelovskii, Matyushkin, and Trishin [59, 60], who carried out a numerical study, and by Kedrinskii [61] and Kinelovskii [62], who obtained an approximate analytical solution to the problem of the convergence of a ring of ideal incompressible liquid. Comparison with experiment showed that the starting stage of the process, on the basis of this model, is described satisfactorily.

Of course, the model of an ideal liquid could not explain a number of interesting physical effects observed in experiments: stopping of the casing accelerated by an explosion and explosive vaporization of its inner layers in the region close to the axis (Fig. 11) shows the process of simultaneous collapse of a copper cylindrical shell 1 and its "explosive" decomposition 2). The explanation was given by Matyushin and Trishin [63, 64] within the framework of the model of a viscous liquid: as a result of irreversible losses of part of the kinetic energy of the collapsing ring its inner layers can be heated up to the temperature of vaporization and form the experimentally observed high-velocity jet (it is known that for beryllium shells its velocity reached 90 km/sec).

The problems of welding by explosion — a phenomenon discovered by Bichenkov, Deribas, Sedykh, and Trishin [65] under conditions of high-velocity interaction of plates — stimulated interest in classical problems of asymmetric collision of flat jets, which, as is well known, do not have a unique solution. A definite step in this direction was taken by Kinelovskii and Sokolov [66]. Based on the results of experimental and numerical studies they formulated the hypothesis that amongst all possible flow configurations the configuration realized is the one for which the curvature of the section of the branching stream line in receding jets is minimum. Trishin [67], based on an analysis of the position of the centers of inertia of liquid elements, separated on the postcollision converging and diverging jets, and on the condition that in the limit the problem must reduce to the problem of a symmetric collision, obtained an analytic solution for the masses of the diverging jets:

$$\frac{m_1^*}{m_2^*} = \frac{1 - \cos\alpha/(1 - \mu^2 \sin^2\alpha)}{1 + \cos\alpha/(1 - \mu^2 \sin^2\alpha)},$$

where $\mu = (m_2 - m_1)/(m_2 + m_1)$ determines the ratio for the masses of the converging jets (the index 1 refers to the thinner jet). In addition it turned out that if 2α is the angle between the converging jets, then $2\alpha = \varphi + \psi$, where φ and ψ are the angles of the diverging jets relative to jet 1.

One of the important practical problems is the problem of the penetration of a jet into a barrier. In this case the densities of the jet and the barrier can be different, which imposes specific features on the character of the flow. According to [48], for the case when the constants in Bernoulli's integral are equal, the entire flow can be described by continuous analytical functions (the line separating the flows γ passes through the critical point). Kinelovskii and Trishin [68] analyzed this type of flow in detail for the example of a symmetric collision of flat two-layer jets of ideal incompressible liquid for different constants in Bernoulli's integral in the layers. They showed that for some regimes of the stationary potential flow for close values of the constants and small relative thickness of the outer layer, the approximate solution is obtained by well-known methods and agrees completely with the experimental results.

Jet Flows Accompanying Underwater Explosions. In the 1970s, under the initiative of Lavrent'ev, a series of experimental and theoretical studies of nonstationary flows with free boundaries, which essentially opened up in the hydrodynamics of explosions a new class of nonstationary jet flows arising with underwater explosions near bounding surfaces, was carried out at the Institute of Hydrodynamics of the Siberian Branch of the USSR Academy of Sciences.

Lavrent'ev [69] examined a paradoxical effect arising in the problem of the destruction of a barrier by a noncontact underwater explosion: it was observed that there exists an appreciable interval of distances away from the wall at which the amount of explosive required to destroy the wall is constant. He called attention to the local character of the destruction and proposed the following model for the process: the presence of the solid wall, by virtue of the asymmetry of the flow, distorts the form of the explosion cavity in the process of the collapse of the cavity, and this leads to the formation of a high-velocity cumulative jet, which is directed toward the wall and can destroy the wall when it impinges against it.

A wide class of jet flows was discovered and investigated by Kedrinskii [70-74] in an analysis of vertical plumes on the free surface of a liquid with underwater explosions. The plumes were distinguished by the directed nature of the ejection, characteristic only for a liquid and not occurring for analogous explosions in soils. Lavrent'ev participated directly in the discussion of the formulation of the problems and the results, and he proposed the first physically consistent model of the process [69]: the shock wave from the explosion forms on the free surface a cumulative depression, and the expanding explosion cavity creates a velocity field orthogonal to its surface. Flow into the depression leads to development of a cumulative jet. Another model, which is useful for large-scale explosions, was proposed by Ovsyannikov [75] on the basis of the results of the solution of an exactly formulated problem of the floating up of a bubble. According to this model, the explosion cavity with detonation products, which has a maximum size, is deformed in the process of floating up in such a manner that an upward directed cumulative jet forms at the bottom of the cavity. It is conjectured that for an appropriate depth of the explosion this effect can determine the structure of one of the directed ejections.

Experimental and numerical investigations carried out by Kedrinskii [70, 72] made it possible to determine the mechanism of the development of plumes and their structure and to construct a simple hydrodynamic model of the phenomenon. He proved that the basis for the structure of the vertical ejection for the first group of explosion depths ($h < R_{max}$) forms a jet tandem, the first jet of which is formed as a result (Fig. 12a) of inertial motion of the layers of liquid above the explosion cavity after negative radial accelerations appear at the cavity, and the second jet of which forms as a result of the closure of the

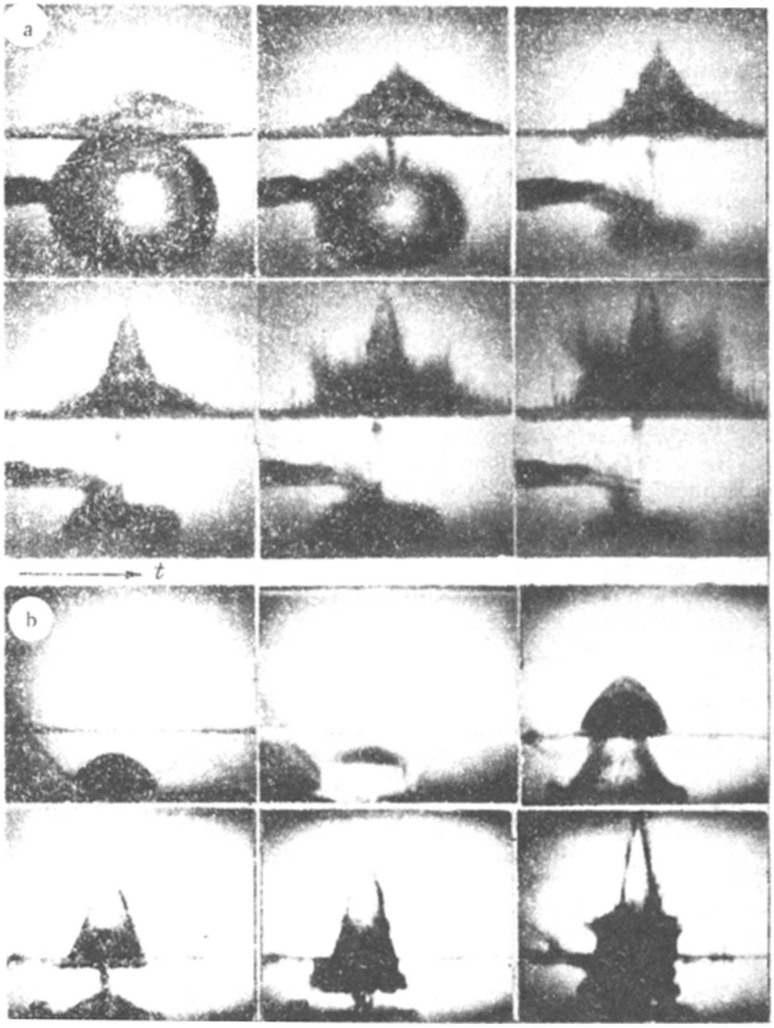

Fig. 12

open cavern, formed after the explosion cavity is opened up. It turned out that the cumulative depression mentioned above does not play a significant role in the formation of the first jet, while its mechanism is clearly demonstrated by the development of the jet flow as a result of projection of the layer of liquid on the initially flat free surface by a spherical or cylindrical solid body (Fig. 12b). It was found experimentally that if the tandem formation process is reversed, then the structure of the flow will be adequate for surface effects developing under conditions of high-velocity penetration of a bullet into water.

For explosion depths of the order of R_{max} the vertical plumes vanish. It was found experimentally and later computationally that at the collapse stage the explosion cavity forms a vertical high-velocity cumulative jet, directed away from the free surface into the bulk of the liquid and destroying the connectivity of the flow region [70]. Penetration of this jet into the liquid explains the experimentally observed anomalous increase in the pressure near the point of collapse. Numerical calculations showed that, at the next stage of the expansion of the explosion cavity separated by the jet, radial (lateral) jet flows, clearly observed in the experiments, form on the free surface.

As the explosion depth is further increased, vertical ejections, which also have a jet structure and whose formation mechanism is described by the model of Ovsyannikov [75], arise once again. It is shown in [74] that there exists a different type of vertical plume, which develops with an explosion of a ring-shaped charge consisting of DSh near a free surface. These experiments essentially modelled one other possibility for the appearance of directed ejection under conditions of large-scale underwater explosions at large depths. In this case the explosion cavity transforms, as it rises, into a stable torus with detonation products, whose pulsation can lead to the indicated effect.

Amongst studies of surface effects we should call attention to two works on the problem of modeling of an explosion on a free surface. Based on investigations of eddy dynamics Deribas and Pokhozhaev [76] concluded that the flow arising is self-similar, and the parameter determining the effect of the explosion is the momentum imparted to the liquid. Accurate experiments, performed by Minin [77], clarified the fact that the law governing the development of the eddy, obtained in the preceding work, does not correspond to the motion of a weightless liquid. The self-similarity remains, however, and its indicator equals 0.47 and 0.38 for cylindrical and point explosions, respectively. According to the data of [76], it equals 0.3.

Explosions in Soils. In this section we do not pretend to give an analysis of the general status of work on explosions in soils and rocks over the last thirty years, especially since this was done in the review in [78] for data up to the beginning of the 1970s. The main attention is devoted to those directions and new problems in this area of science which are most closely associated with Lavrent'ev's school. Here three directions can be conditionally distinguished, noting also the dynamics of deformation of soils, for which more accurate experimental studies have made it necessary to take into account the effect of viscosity and dilatancy accompanying an explosion [79-82], penetration of detonation products into the soil, and concomitant heat-transfer processes [83].

Theory of Camouflet Explosions. The main theoretical study of the dynamics of the medium under the conditions of a camouflet explosion are zone models, developed in 1950-1970 [78] and constructed based on a different description of the behavior of the medium depending on the strength of the effect of the explosion load on the medium.

Amongst the works performed by Siberian scientists here we call attention to the investigations of E. I. Shemyakin [84, 85], devoted to the determination of the dynamics of stress waves in solids under the conditions of a camouflet explosion. Comparison of the experimental data on the attenuation of diverging stress waves with the asymptotic behavior, obtained in the explosion problem by the method of short waves for a medium with friction, showed that for a large range of distances the asymptotics describe an experiment well, if it is assumed that the plastic state $\sigma_r = \alpha\sigma_\theta + \beta$ and $\alpha = \nu/(1 - \nu)$, where ν is Poisson's ratio in the elastic state of the medium.

The quite interesting, for the practical viewpoint, problem of a camouflet explosion permits checking different modifications of models and evaluating by means of a numerical experiment the effect of various factors. In this manner models were constructed and calculations of the dynamics of the medium with a camouflet explosion were carried out taking into account viscosity [86, 87], dilatancy [88, 89], the nonadiabatic nature of the detonation products and heat transfer as the detonation products penetrate into the porous medium [90-92].

There are a number of works on the experimental study of the granular composition of the pieces into which the rock is fragmented by an explosion. Kuznetsov and his students [93, 94] successfully employed the Rozin–Ramler distribution to describe the granular composition. Data on the dependence of the lumpiness parameters on the distance are presented in [95].

The model describing the destructive action of the explosion on rocks was improved. The new feature here is the application of the mechanics of brittle fracture (Sher, Cherepanov, and others), enabling the evaluation of the dynamics of cracks [96-99] and description of the expected degree of destruction of the mass after the explosion. There are many works on the description of the change in the filtrational permeability of rocks under the action of an explosion [100].

Excavation Blasts: Experiment and Hydrodynamic Models. Here attention was devoted to the problems of improving the technology of blasting, in particular, the development of channels using a series of point charges and networks of well charges; more accurate empirical dependences were determined for the parameters of the funnel on the geometry of the arrangement and mass of the charge [101, 102]; and, the effect of water-saturated levels on the formation of the blasting crater was analyzed [103]. The method of modeling large explosions [89, 104], making it possible to model the soil dispersion stage taking into account gravity, has been substantially developed.

The ultimate goal of studies of the action of an explosion, associated with the development of hydrodynamic models, is to solve the practical problems of determining the parameters of the blasting crater for realistic configurations of explosive charges taking into account free surfaces and the characteristics of the soil structure (stratification, the presence of water-bearing strata, etc.).

The hydrodynamic models for describing the dynamics of solid media under the action of intense explosive loads are based on the "incompressibility scheme" proposed by Lavrent'ev [78]. Based on this scheme the result of an explosion is evaluated from the velocity field v arising in the medium as an incompressible liquid after the momentum from the explosion is transferred to it. To describe the velocity the following problem is posed: $\Delta\varphi = 0$, $\bar{v} = \operatorname{grad}\varphi$ and at the boundary the potential is determined by the integral $\varphi = -\frac{1}{\rho}\int\limits_0 p(t)\,dt.$

According to this model the velocity field in the soil from an explosion of a buried string charge with a small radius (compared with the depth h), parallel to the free surface, is described by the complex potential of the source with intensity q, lying at a distance h from the free surface, where $\varphi = 0$:

$$w = \frac{q}{2\pi}\ln\frac{z+ih}{z-ih}.$$

Here φ = Re w and the velocity on the free surface has the form

$$v_y = \left.\frac{\partial\varphi}{\partial y}\right|_{y=0} = \frac{qh}{\pi(x^2+h^2)}.$$

It is maximum above the charge and decays as $1/x^2$ as $x \to \pm\infty$.

As suggested by Vlasov and Lavrent'ev the boundary of the crater on the free surface is found from the condition $v_y = c$ (c is the critical velocity characterizing the strength of the soil). In the practical application of such calculations it is necessary to know the parameters q for the given charge and c for the soil. For this purpose there exist definite dependences, which include empirical constants, determined with the help of calibration explosions [105, 106].

Within the framework of the liquid model it is possible to solve "inverse" problems, of which the problem of a directed explosion is important in practice: to find the distribution of the explosive on the surface of a given volume of soil that provides after the explosion a motion with a constant velocity over the volume. It was first solved by Lavrent'ev, Kuznetsov, and Sher, who showed that in this case the indicated distribution must provide a linear flow potential in the given volume [107]: $\varphi = ax + by + cz + d$.

Transformation from the potential to the distribution density of the explosive charge is carried out taking into account the conditions for the operation of the charge. If the charge operates like a superposed charge without stemming, then it may be assumed that the momentum from it is proportional to its thickness, and the distribution density of the charge over the surface of the volume is proportional to the potential and varies linearly with the distance in the direction of ejection.

According to the liquid model the soil is regarded as a liquid, which, on the one hand, simplifies the problem of finding the velocity field, but on the other does not permit determining the entire profile of the blasting crater: only its width is calculated. This problem is solved based on the new solid—liquid model proposed by Lavrent'ev and first implemented by Kuznetsov [105]: near the charge, where the velocity is greater than some critical value c, the soil is regarded as a liquid, and in the rest of the region it is regarded as an absolutely rigid body. The boundary of the crater is not known beforehand and is sought as a streamline, on which the velocity $|v| = c$. We note that the problem of the form of the blasting crater in this formulation is more complicated and belongs to the class of "stream" problems in the theory of flow around objects.

In the case of the explosion of a deep string charge (the physical formulation is analoggous to that described above) the problem was solved by Martynyuk [108]. For this it is possible, in particular, to use the method of singularities. The introduction of the auxiliary plane of the complex variable ξ with the correspondence, indicated in Fig. 13a, between it and the plane of the real flow z, yields the following expression for the complex potential

$$w(\xi) = \left(\ln\frac{\xi-b}{\xi+b} + \ln\frac{1-\xi b}{1+\xi b}\right)q/2\pi$$

its derivative

$$\frac{dw}{d\zeta} = a(1-\zeta^2)(\zeta^2-b^2)(1-\zeta^2b^2)$$

and the complex velocity

$$\frac{1}{v}\frac{dw}{dz} = i(1-\zeta^2b^2)(\zeta^2-b^2).$$

From here we have

$$v\frac{dz}{d\zeta} = a(1-\zeta^2)(1-\zeta^2b^2)^2, \quad a = \frac{qb}{\pi}(1-b^2).$$

Then for the shape of the crater CD we have

$$\frac{2\pi v z}{q} = \frac{(1+b^2)^2}{2b^2}\ln\frac{1+\zeta b}{1-\zeta b} - \frac{1+b^4}{b}\frac{\zeta}{1+\zeta^2b^2}. \tag{9}$$

The parameter b here is determined from the condition z(b) = −ih (the point at which the source is located) using the equation

$$\frac{(1+b^2)^2}{2b^2}\ln\frac{1+b^2}{1-b^2} - 1 = \frac{2\pi v h}{q}.$$

On the basis of the scheme indicated, starting with his first work [109] Kuznetsov and his students solved many problems of practical interest concerning explosions in a layer with a solid foundation, in a two-layer medium, the detonation of interacting charges, etc. [105]; some configurations, currently under study, are shown in Fig. 13b, where 1 is the explosive charge, 2 is the free surface, and 3 is the boundary of the layers of soil with different critical velocity. The class of solutions obtained with the help of this model was significantly expanded by the school of mathematicians in Kazan headed by Il'inskii [106].

An important practical question is the correspondence between the theoretical solutions and the experimentally determined craters. Analysis of different formulations, carried out in [105, 106], showed that both models mentioned correctly predict many qualitative aspects of the change in the parameters of the blasting craters as a function of the geometry of the arrangement of the charges. However disagreements are also observed. Thus, Polyak and Sher [110] found that the experimental craters from the explosion of an overlayed charge are much shallower than according to (9), and that this effect depends strongly on the internal friction angle of the soil. To describe this phenomenon they proposed a modified solid–liquid model, in which the condition at the boundary of the crater was changed: $|v| = c + k\varphi$. Sher and Perminov [111] confirmed that by varying the parameters c and k it is possible to obtain theoretical crater profiles which are close to the experimental profiles for an explosion of a buried string charge also. Here, however, it is now necessary to determine three constants (q, c, and k) in terms of the properties of the soil and parameters of the charge. In addition, to predict the possibilities of the proposed model the results of a large number of experimental explosions must be analyzed.

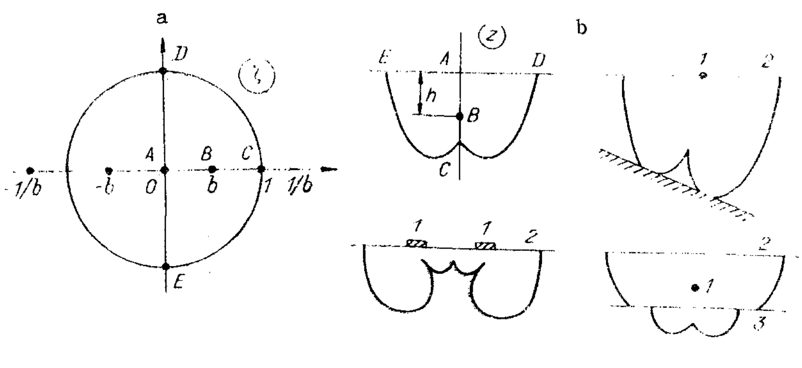

Fig. 13

Hydrodynamic Models of the Destructive Action of an Explosion. The determination of the zones of destruction in rock under the action of an explosion and evaluation of the size spectrum of the crushed rock is one of the important practical problems. The complexity of this problem forces investigators to use simple hydrodynamic models. Although the use of such models in questions of the fracture of solids has not yet been justified, the results obtained often are confirmed in practice. This paradox was noted by Valsov and Smirnov, who first developed such a model [112], which was somewhat improved later by Kuznetsov [105]. Formally it is similar to the liquid model of an excavation blasting: the pulsed velocity field in an ideal incompressible liquid, generated by the explosive load, is sought; the boundary of the region of fracturing is found from the critical velocity c; and, the size of the pieces is determined by the intensity of the shear velocities.

The problem of the distribution of the explosive charge on the boundary of the volume of a solid body, whose detonation destroys the given body uniformly over the volume, was solved on the basis of this model. The solution gives the distribution of the explosive charge that provides a quadratic distribution of the potential. In the two-dimensional case, there arises here a flow with the complex potential $\omega = a + bz^2$. Based on the solution, Kuznetsov and Sher [105, 113] proposed a method for uniform shear fracture for constructing practical grids for drilling-blasting work in quarries.

Shock Waves in Layered Systems. The problems of the hydrodynamics of explosions also involve the questions of the propagation and cumulation of shock waves in layered systems, closely related with the problems of explosion welding, high-velocity projection cleavage, etc. An example of unbounded cumulation in the two-dimensional case was first constructed by Zababakhin [114], who studied the motion of a wave with a front parallel to alternating flat layers of light and heavy materials. If each heavy (light) layer is thinner than the preceding layer, the shock wave is intensified, this idea was confirmed experimentally by Kozyrev et al. [115] in a scheme consisting of layers of plexiglass and lead.

A new principle for the construction of layered systems was proposed by Trishin and Laptev [116]. It is based on the characteristics of the propagation of SW in a medium with a gradient of the acoustic impedance R. The fact that the velocity of the wave is oriented along the gradient leads to pressure cumulation, while the opposite orientation leads to an increase in the mass velocity, which can exceed the starting velocity of the striker, generating the wave. The acoustic approximation permitted the authors to obtain corresponding analytical dependences. The problem for strong SW in the case $R = \rho_0 D = \rho_0(a + bu)$ was studied by Kroshko and Chubarova [117], who obtained analogous results in a computer experiment. Some aspects of the propagation of strong SW in layered systems of this kind were analyzed numerically, taking into account the nonlinear interaction, by Sapozhnikov and Fomin [118], who confirmed unequivocally the relationship between cumulation effects and the change in the acoustical properties of the medium. Nestrenko [119] pointed out that cumulation of a wave is observed not only because of its interaction with the contact discontinuities, but also as a result of compression waves overtaking the head wave. This question was investigated in detail by Fomin, Nesterenko, and Cheskidov.

Trishin and Fomin [120] examined gaseous systems with a discrete distribution of the density in the layers. In their experiments, analogous to those of Voitenko et al. [121], for a starting pressure differential in the layers equal to 16, an increase of the mass velocity by more than 40% behind the SW front was obtained. Ternova [122], based on the principle mentioned above [116], recorded the projectile velocity of plates set in motion by an explosion exceeding 10 km/sec. For projection of solid layered strikers with metallic cumulative jets [123] velocities of the order of the velocities of the head sections of cumulative jets have been achieved.

For high-velocity colisions in solids there arise SW with a quite high amplitude, which can give rise to, in particular, cleavage phenomena and can affect, for example, the quality of the joint in explosive welding. Kachan, Kiselev, and Trishin [124-128] called attention to the fact that the collection of plates being welded, the supports, and the interlayers between the explosive charges and the striker plate form a layered system, and they employed previously obtained solutions for the parameters of the SW and the rarefaction waves to analyze the effect of its components on the strength of the bond. They emphasized, in particular, that the acoustic stiffness of the interlayer and the immobile plate must be less than that of the striker plate. In [128] it was demonstrated that cleavage phenomena in collision problems can be controlled by constructing layered systems which vary according to a definite law.

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